Eur. Phys. J. D **37**, 327–334 (2006)
DOI: 10.1140/epjd/e2005-00328-9

THE EUROPEAN PHYSICAL JOURNAL D

Phase (or gauge) invariance and new field-free Aharonov-Bohm effects

G. Spavieri[a]

Centro de Física Fundamental, Facultad de Ciencias Universidad de Los Andes, Mérida 5101, Venezuela

Received 15 September 2005
Published online 13 December 2005 –

Abstract. The criteria for phase invariance of quantum effects of the Aharonov-Bohm (AB) type are revised. Gauge invariance, duality and other properties of the electromagnetic interaction lead to new field-free quantum effects of the AB type for interfering beams of particles with opposite charges or magnetic dipole moments.

PACS. 03.30.+p Special relativity – 39.20.+q Atom interferometry techniques – 03.65.Bz Foundations, theory of measurement, miscellaneous theories (including Aharonov-Bohm effect, Bell inequalities, Berry's phase)

1 Introduction

In quantum effects of the Aharonov-Bohm type (AB) [1–8] a beam of interfering particles interacts with an external potential or field. The Lagrangian has the usual form $\mathfrak{L} = K - V + \mathbf{v}\cdot\mathbf{Q}$, where $\mathbf{Q}$ is the electromagnetic (em) interaction momentum and the quantum phase of the particle for the corresponding Hamiltonian H reads $\phi = \hbar^{-1}\int \mathbf{Q}\cdot d\vec{\ell}$.

Effects of this kind are relevant in a wide context of scientific and heuristic situations, such as those involving quantum non-locality and topology [1–9], electromagnetic interaction and duality [3–10], and even in some tests of the validity of Coulomb's law [11].

Usually, in these effects, the beam of particles is split into two interfering beams that encircle the sources (Fig. 1), or em "singularity" (i.e., the singularity ideally represented by a line of em sources), and are then recombined to produce an interference pattern.

Some of the effects of the AB type have been verified experimentally; for a discussion of them see references [12,13] and, for tests of the Aharonov-Casher (AC) effect, see reference [14]. However, in one recent test of the AC effect [15], the paths of the two beams of particles with opposite em properties (in this case, opposite magnetic dipole moments $\pm\mathbf{m}$) in a coherent superposition of states $|a\rangle$ and $|b\rangle$, were not spatially separated but did lie on the same side of the singularity (Fig. 2). A technique for testing these kinds of quantum effect for electric dipoles $\pm\mathbf{d}$ in a coherent superposition of states $|a\rangle$ and $|b\rangle$ that are not spatially separated and lie at one side of the singularity has also been proposed [10].

[a] e-mail: spavieri@ula.ve

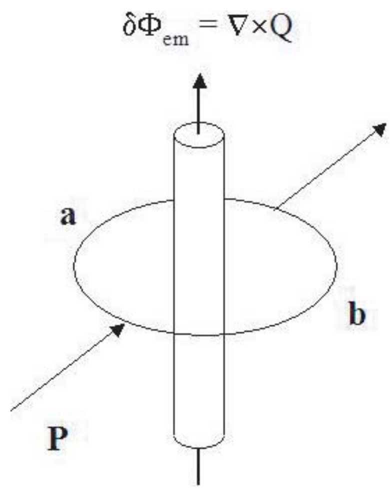

Fig. 1. In most of the effects of the Aharonov-Bohm type, a beam of particles P is split into two beams a and b that encircle the em flux tube $\delta\Phi_{em}$ and form an interference pattern visible on a screen. In the AB effect, the particles are electrons and em flux (the "singularity") is a thin solenoid. In the AC effect, the particles are neutrons possessing an intrinsic magnetic dipole moment $\mathbf{m}$ and the singularity is a charged wire.

In the first part of this paper we consider the observable physical quantity that is measured in experimental tests of the AB effect and revise the related criteria of phase or gauge invariance. As a consequence of our revision we find that, for the physical observable of interest, these criteria can be met in a much wider variety of cases than is generally thought achievable. Some additional theoretical developments and the concomitant properties of the em interaction, discussed in the second part, indicate

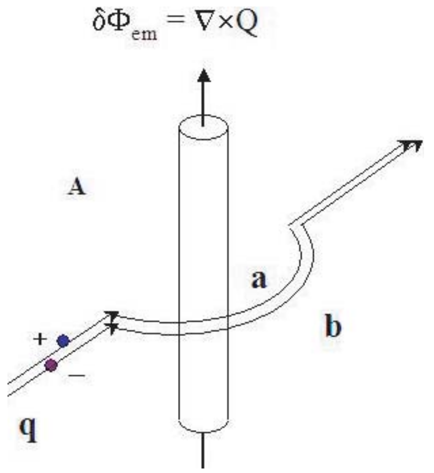

Fig. 2. Quantum effects of the AB type for beams of particles in a coherent superposition of states $|a\rangle$ and $|b\rangle$ that travel at one side of the singular em flux tube $\delta\Phi_{em}$. In the AC effect, the particles possess opposite magnetic dipole moments $\pm\mathbf{m}$. In the field-free effect for the electron-positron Aharonov-Bohm effect, two beams of interfering particles with opposite charges $\pm q$ travel at one side of a solenoid in the presence of the vector potential $\mathbf{A}$ and form an interference pattern visible on a screen. The interference pattern produced in correspondence of $\mathbf{A}$ is compared with the reference pattern corresponding to $\mathbf{A}^0$. The observable effect is given by the relative phase shift $\Delta\phi \propto 2q\int_a(\mathbf{A}-\mathbf{A}^0)\cdot d\mathbf{x})$ and is due to the variation $\Delta\mathbf{Q}_{em} = \mathbf{Q}_{em} - \mathbf{Q}^0_{em} = 2q(\mathbf{A}-\mathbf{A}^0)$ of the interaction em momentum.

that new field-free effects of the AB type, that were considered physically impossible even in principle, are instead viable. The new effects involve beams of particles with opposite electromagnetic properties, such as opposite charge (electron-positron or positively-negatively ionized atoms) or opposite em dipole moments.

2 Phase (or gauge) invariance in the AB effects

What is physically relevant and common to all these quantum effects [7] are the quantities

$$\mathbf{Q}(\mathbf{x}) = \pm\mathbf{Q}_{em} = \pm\frac{1}{4\pi c}\int(\mathbf{E}\times\mathbf{B})d^3x' = L\boldsymbol{\nabla}\theta, \qquad (1)$$

where $\mathbf{Q}_{em}$ is the classical linear momentum of the em fields and $\mathbf{L}$ ($L = |\mathbf{L}|$) is the classical angular momentum of the em fields.

Thus, in the AB effect, $\mathbf{Q}(\mathbf{x}) = \mathbf{Q}_{em} = c^{-1}q\mathbf{A}$, where $\mathbf{A}(\mathbf{x})$ is the vector potential of the thin solenoid, q the charge of the particle, and $\mathbf{x}$ the particle position. The AB term $c^{-1}q\mathbf{A}$ is obtained by taking $\mathbf{E}$ in equation (1) to be the electric field of the charge and $\mathbf{B}$ to be the magnetic field of the solenoid. A general proof that this result holds in the *natural* Coulomb gauge, is given by Boyer [16], Zhu and Henneberger [17], and Spavieri [18]. In the AC effect, $\mathbf{Q}(\mathbf{x}) = -\mathbf{Q}_{em} = c^{-1}\mathbf{m}\times\mathbf{E}$, where $\mathbf{m}$ is the magnetic dipole moment of the particle and $\mathbf{E}$ the external electric field due to a line of charges. For the effects with electric dipoles [5–7], $\mathbf{Q}(\mathbf{x}) = \mathbf{Q}_{em} = c^{-1}(\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{A}$ where $\mathbf{d}$ is the electric dipole moment of the particle.

For all the effects of the AB type the momentum $\mathbf{Q}$ is expressed as $\mathbf{Q}(\mathbf{x}) = L\boldsymbol{\nabla}\theta$, for $r \neq 0$ (where the multivalued function $\theta = \tan^{-1} y/x$), and the geometrical properties of $\mathbf{Q}$ and of the em flux tube $\delta\boldsymbol{\Phi}_{em} = \boldsymbol{\nabla}\times\mathbf{Q} = \mathbf{L}\delta(r)/r$ determine the topology of the phase shift [7]:

$$\begin{aligned}\delta\phi &= \frac{1}{\hbar}\oint_C \mathbf{Q}\cdot d\vec{\ell} = \frac{1}{\hbar}\oint_S(\boldsymbol{\nabla}\times\mathbf{Q})\cdot d\mathbf{S}\\ &= \frac{L}{\hbar}\oint d\theta = 2\pi n\frac{L}{\hbar} = \frac{1}{\hbar}\Phi_{em}.\end{aligned} \qquad (2)$$

If the paths of the beams lie at one side of the singularity (Fig. 2), as in the case of the test mentioned above for the AC effect [15], then

$$\begin{aligned}\delta\phi &= \frac{1}{\hbar}\left(\int_a \mathbf{Q}\cdot d\vec{\ell} - \int_b \mathbf{Q}\cdot d\vec{\ell}\right)\\ &= \frac{2}{\hbar}\int_a \mathbf{Q}\cdot d\vec{\ell} = \frac{1}{\hbar}\oint_C \mathbf{Q}\cdot d\vec{\ell}.\end{aligned} \qquad (3)$$

At this point, one might question the validity of equations (2) and (3) when $\mathbf{Q}$ is expressed in terms of the vector potential $\mathbf{A}$. In fact, in a discussion about gauge invariance, one may argue that in this case $\delta\phi$ is a gauge dependent quantity that can be made to vanish. That phase factors cannot always be removed by a suitable phase or gauge transformation was shown by Berry [19]. To verify that $\delta\phi$ is not a pure phase, it is sufficient to check that [20], if the particle is transported along the closed curve C, it encloses a nontrivial em flux Φ_{em}. Thus, the phase shift of the standard AB effect for a beam of charges encircling the singularity (Fig. 1), given by equation (2) where the integral $\oint_C \mathbf{Q}\cdot d\vec{\ell} = (q/c)\oint_C \mathbf{A}\cdot d\vec{\ell}$ is over the closed curve C, is not removable by a gauge transformation. However, the AB effect for a beam of charges of opposite signs $\pm q$ in a coherent superposition of states $|a\rangle$ and $|b\rangle$, passing on one side of the solenoid (Fig. 2), should not be detectable because

$$\Delta\phi = \frac{2}{\hbar}\int_a \mathbf{Q}\cdot d\vec{\ell} = \frac{2\,q}{\hbar\,c}\int_a \mathbf{A}\cdot d\vec{\ell}$$

can be gauged to zero. This argument is based on the well known theorem that in a simply connected field-free region it is possible to choose a gauge in which the vector potential vanishes identically throughout that region. The vector potential, being curl-free, may be written as the gradient of a scalar function. If one subtracts the gradient of that function from $\mathbf{A}$, one obtains a new $\mathbf{A}'$ which vanishes. Thus, it may be claimed that the mentioned AB effect for $\pm q$ is not observable.

In this section, we revise in points (1) and (2) below, the criteria for phase or gauge invariance in the effects of the AB type. For this, we use the dipole approximation with nonrelativistic velocities, and we neglect the radiation terms of order c^{-2} or higher.

A gauge transformation is a special case of a phase transformation. If the total time derivative of an arbitrary scalar function $\chi(\mathbf{x},t)$, $(d/dt)\chi = \partial_t\chi + \mathbf{v}\cdot\boldsymbol{\nabla}\chi$, is added to the Lagrangian $\mathfrak{L}$, the equations of motion do not change because

$$\left(\frac{d}{dt}\frac{\partial}{\partial \mathbf{v}} - \boldsymbol{\nabla}\right)\frac{d}{dt}\chi = \frac{d}{dt}\boldsymbol{\nabla}\chi - \boldsymbol{\nabla}(\partial_t\chi + \mathbf{v}\cdot\boldsymbol{\nabla}\chi) = -\mathbf{v}\times(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\chi) = 0,$$

while the wave function for H acquires the pure phase $\hbar^{-1}\int \boldsymbol{\nabla}\chi\cdot d\mathbf{x} = \hbar^{-1}\chi(\mathbf{x},t)$. However, since $\oint_S(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\chi)\cdot d\mathbf{S} = 0$, there is no em flux associated with the term $\boldsymbol{\nabla}\chi$.

Special care should be given to the case of particles with opposite em properties in a coherent superposition of states $|a\rangle$ and $|b\rangle$ that travel (for example, on path a) at one side of the singularity. Since we have $\boldsymbol{\nabla}\times\mathbf{Q} = 0$ along this path, the quantity $\mathbf{Q}$ may be expressed as the gradient of a scalar function and a suitable phase transformation may be chosen such that $\int_a(\mathbf{Q} - \boldsymbol{\nabla}\chi)\cdot d\vec{\ell} = 0$. Thus, even the above-mentioned AC effect, wherein the paths of the two beams of particles are not spatially separated and lie at one side of the singularity [15], should not be detectable. However, the result of the measurement of the AC phase shift of this effect [15] indicates that $\Delta\phi = \hbar^{-1}2\int_a \mathbf{Q}\cdot d\vec{\ell}$ does not vanish and is an observable physical quantity that cannot be removed by a phase transformation.

The following arguments clarify the problem of phase (or gauge) invariance in AB effects.

2.1 Phase (or gauge) invariance and the observable phase shift variation

In the experimental test of these effects the reference pattern of two interfering wave functions Ψ_a^0 and Ψ_b^0, with phase difference $\delta\phi^0 = \phi_a^0 - \phi_b^0$ (corresponding to $\mathbf{Q}^0$) and of intensity

$$I_{ab}^0 = I_a^0 + I_b^0 + 2\sqrt{I_a^0 I_b^0}\cos(\delta\phi^0),$$

is created. This pattern, with its characteristic fringes, is observable and visible on a screen. The interference pattern of the wave functions Ψ_a and Ψ_b, of intensity

$$I_{ab} = I_a + I_b + 2\sqrt{I_a I_b}\cos(\delta\phi)$$

and phase difference $\delta\phi = \phi_a - \phi_b$ (corresponding to $\mathbf{Q}$), is compared with the reference pattern and the relative pattern displacement ε, a shift visible on the screen, is measured. Hence, although it is not possible to determine independently the value of $\delta\phi$, or $\delta\phi^0$, the relative phase shift $\delta\phi - \delta\phi^0$ can be measured, since it is related to the relative displacement ε of the patterns, or fringes, observable on the screen.

Thus, the observable quantity that is actually measured in these effects is the phase shift variation

$$\Delta\phi = \delta\phi - \delta\phi^0 = \frac{2}{\hbar}\int_a \mathbf{Q}\cdot d\vec{\ell} - \frac{2}{\hbar}\int_a \mathbf{Q}^0\cdot d\vec{\ell} = \frac{1}{\hbar}\oint_C(\mathbf{Q} - \mathbf{Q}^0)\cdot d\vec{\ell}, \tag{4}$$

where usually $\mathbf{Q}^0 = 0$. For example, to test the AB effect with a solenoid, a tapering iron whisker was used [12] so that its magnetic flux varies along its length. The AB relative shift $\Delta\phi_{AB}$ was observable by comparing the relative position of the sets of fringes displaced, or tilted, by the varying magnetic flux of different segments of the whisker. In the measurement of the AB effect with a toroid [13], two waves — the object wave due to the sample (toroidal magnet) and the reference wave — form an interference pattern with phase shift $\delta\phi$ inside the toroid which is reconstructed and compared with the interference reference pattern with phase shift $\delta\phi^0$ outside the toroid. From the relative position or displacement ε of the fringes, the quantity $\delta\phi - \delta\phi^0$ is determined. Similarly, in the measurement of the AC effect [15], the phase shift variation $\delta\phi - \delta\phi^0$ is obtained by changing the sign of the applied external $\mathbf{E}$ field (this implies $\mathbf{Q}^0 = -\mathbf{Q}$ in Eq. (4)).

If a phase transformation is performed for the mentioned effects, the quantity $\Delta\phi$ of equation (4) transforms as

$$\Delta\phi' = \frac{2}{\hbar}\int_a(\mathbf{Q} - \boldsymbol{\nabla}\chi)\cdot d\vec{\ell} - \frac{2}{\hbar}\int_a(\mathbf{Q}^0 - \boldsymbol{\nabla}\chi)\cdot d\vec{\ell} = \frac{2}{\hbar}\int_a(\mathbf{Q} - \mathbf{Q}^0)\cdot d\vec{\ell} = \Delta\phi, \tag{5}$$

i.e., the observable $\Delta\phi$, measured in these quantum effects, is phase invariant. Each one of the two terms (the integrals) on the rhs of equation (5), is not an observable quantity and, taken separately, can be set to zero by a phase transformation. However, their difference $\Delta\phi' = \Delta\phi$ is an observable physical quantity that cannot be gauged (or phase-transformed) to zero.

2.2 The observable phase shift variation and the em forces

The following argument, which is an elaboration of that discussed in a previous work [8], corroborates the result of point (1).

The phase shift variation $\Delta\phi$ generally relates to force-free quantum effects that have no classical origin and do not involve measurements with time-varying fields or potentials. The reference pattern corresponding to $\delta\phi^0$ is created using a beam of particles with the fixed, time-independent sources of potentials and fields producing $\mathbf{Q}^0$, and the pattern corresponding to $\delta\phi$ is created using a beam of particles with the fixed sources of potentials and fields producing $\mathbf{Q}$. Thus, we are discussing here field-free or force-free quantum effects and it should be clear that there are no classical forces acting on the particles during the process of measurement when the fixed pattern corresponding to $\delta\phi^0$ and the fixed pattern corresponding to $\delta\phi$ visible on the screen are compared.

However, if $\delta\phi^0$ (and $\mathbf{Q}^0$) is thought of as being brought to the value $\delta\phi$ (and $\mathbf{Q}$) by ideally varying the sources of fields and potentials, there would be nonvanishing em forces $\mathbf{f}$ that may potentially act on the particles. For example, if in the AB effect the current in the solenoid is switched on and the vector potential increases from the value $\mathbf{A}^0 = 0$ (and $\mathbf{Q}^0 = 0$) to the finite value $\mathbf{A} \neq 0$ (and $\mathbf{Q} \neq 0$), then there is an em force $\mathbf{f} = q\mathbf{E} = -(q/c)\partial_t \mathbf{A}$ that may act on a nearby charged particle q while $\mathbf{A}$ varies. Thus, there may exist a relationship between the resulting phase shift variation $\Delta\phi$ and the hypothetical em force $\mathbf{f}$ that could act on the beam of particles during the time that the sources of fields and potentials vary. In order to find this relationship we use a semi-classical argument and proceed as follows.

The equations of motion for $\mathfrak{L}$ read

$$\mathbf{f} = \frac{d}{dt}(m\mathbf{v}) = -\frac{d}{dt}\mathbf{Q} + \boldsymbol{\nabla}\mathfrak{L} = -\boldsymbol{\nabla}V - \partial_t \mathbf{Q} + \mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{Q}), \tag{6}$$

where outside the singularity $\boldsymbol{\nabla}\times\mathbf{Q} = 0$. For a beam or distribution of particles along a path a of length l_p, let us consider the force density $\overline{\mathbf{f}} = \mathbf{f}/l_p$. Taking for simplicity $l_p = 1$, the initial phase $\phi = \hbar^{-1}(\mathbf{p}\cdot\mathbf{x} - E\,t)$ of a free non-interacting beam of particles is shifted by the amount

$$\delta\phi = \hbar^{-1}\int_a \delta\,\mathbf{p}\cdot d\mathbf{x} = \hbar^{-1}\int_a \delta\,(m\mathbf{v})\cdot d\vec{\ell} = \int dt \int_a \mathbf{f}\cdot d\vec{\ell}$$

by the action of the force $\mathbf{f}$. For the relative change $\int_a \mathbf{f}\cdot d\vec{\ell} - \int_b \mathbf{f}\cdot d\vec{\ell} = 2\int_a \mathbf{f}\cdot d\vec{\ell} = \oint_C \mathbf{f}\cdot d\vec{\ell}$, we have from equation (6)

$$\oint_C \mathbf{f}\cdot d\vec{\ell} = -\frac{d}{dt}\oint_C \mathbf{Q}\cdot d\vec{\ell} = -\frac{d}{dt}\oint_S \boldsymbol{\nabla}\times\mathbf{Q}\cdot d\mathbf{S} = -\frac{d}{dt}\Phi_{em}, \tag{7}$$

where $\delta\boldsymbol{\Phi}_{em} = \boldsymbol{\nabla}\times\mathbf{Q}$ is the flux tube density and Φ_{em} the em flux through the surface S. The integration of (7) over dt, from $t = 0$ to t with $\mathbf{Q} = \mathbf{Q}(t)$ and $\mathbf{Q}^0 = \mathbf{Q}(t = 0)$, yields

$$-\int_0^t dt \oint_C \mathbf{f}\cdot d\vec{\ell} = \oint_C (\mathbf{Q} - \mathbf{Q}^0)\cdot d\vec{\ell} = \oint_S \boldsymbol{\nabla}\times\mathbf{Q}\cdot d\mathbf{S} = \Phi_{em}(t) = \hbar\,\Delta\phi. \tag{8}$$

For simplicity in equation (8) we have assumed that $\mathbf{Q}^0 = 0$ ($\Delta\phi = \delta\phi$ and $\Delta\Phi_{em} = \Phi_{em}$) as usual. However, the term $\mathbf{Q}^0$ must be retained formally when checking for phase or gauge invariance.

Since $2\int_a \mathbf{f}\cdot d\vec{\ell} = \oint_C \mathbf{f}\cdot d\vec{\ell}$, for the case of two beams of particles in a coherent superposition of states $|a\rangle$ and $|b\rangle$ not spatially separated that lie at one side of the singularity, equation (8) reads

$$-2\int_0^t dt \oint_a \mathbf{f}\cdot d\vec{\ell} = 2\int_a \mathbf{Q}\cdot d\vec{\ell} = \oint_C \mathbf{Q}\cdot d\vec{\ell} = \Phi_{em}(t) = \hbar\,\Delta\phi. \tag{9}$$

As mentioned above, the phase shift variation $\Delta\phi$ generally relates to force-free quantum effects that have no classical origin and do not involve measurements with time-varying fields or potentials. In the actual tests of the AB effects, the reference pattern corresponding to $\delta\,\phi^0$ is created using a beam of particles with the fixed sources of potentials and fields producing $\mathbf{Q}^0$, and the pattern corresponding to $\delta\,\phi$ is simultaneously created using a beam of particles with the fixed sources of potentials and fields producing $\mathbf{Q}$. However, if $\delta\,\phi^0$ is brought to the value $\delta\,\phi$ by ideally varying the sources of fields and potentials, the reference pattern shifts itself on the screen until it coincides with the other pattern. In this hypothetical case the corresponding phase shift variation $\Delta\phi$, which at the end is always the same, may be related to $\mathbf{f}$ by means of (8), or (9), which states that $\hbar\,\Delta\phi$ is attributable to the phase variation, associated with the change $\delta\,(m\mathbf{v})$ of the linear momentum accumulated along the classical path, produced by the hypothetical em force $\mathbf{f}$ that would arise if the sources of the fields and potentials were made to vary.

The expressions (4), (5), (8) and (9) for $\Delta\phi$ are completely equivalent from a physical point of view. Equations (4) and (5) express the value of $\Delta\phi$ for the force-free effects of the AB type in terms of the static variation $\mathbf{Q} - \mathbf{Q}^0$. Equations (8) and (9) express the value of $\Delta\phi$ in terms of the hypothetical force $\mathbf{f}$ that could act on the particles if the variation $\mathbf{Q} - \mathbf{Q}^0$ were to occur in time. However, the advantage of deriving equations (8) and (9) is that the lhs is expressed in term of an invariant quantity, the force $\mathbf{f}$. Since a pure phase (or gauge) transformation does not modify the fields and forces, even the rhs of equations (8) and (9), where $\mathbf{Q}^0$ has been set equal to zero, is left invariant.

Therefore, the established relationship (9) is another proof that $\Delta\phi$ cannot be removed by a phase or gauge transformation.

Let us check now the validity of equation (9) for the AB effect where $\mathbf{Q}_{AB} = (q/c)\mathbf{A}$. Recalling that on the path a it is $\boldsymbol{\nabla}\times\mathbf{Q} = \boldsymbol{\nabla}\times\mathbf{A} = 0$, the force is $\mathbf{f} = q\mathbf{E} = -(q/c)\partial_t\mathbf{A}$, so that in equation (9)

$$-2\int_0^t dt \int_a \mathbf{f}\cdot d\vec{\ell} = 2\frac{q}{c}\int_a \mathbf{A}\cdot d\vec{\ell} = \frac{q}{c}\oint_C \mathbf{A}\cdot d\vec{\ell} = \hbar\,\Delta\phi_{AB}$$

as expected. For the AC effect $\mathbf{Q}_{AC} = \mathbf{m}\times\mathbf{E}/c$ with $\mathbf{m}$ perpendicular to $\mathbf{v}$ and $\mathbf{E}$. In this case the force on $\mathbf{m}$ reduces to $\mathbf{f} = (\mathbf{m}\cdot\boldsymbol{\nabla})\mathbf{B} - c^{-1}\dot{\mathbf{m}}\times\mathbf{E}$ [4,21,22]. For time-varying fields but $\dot{\mathbf{m}} = 0$, $\mathbf{f} = (\mathbf{m}\cdot\boldsymbol{\nabla})\mathbf{B} = \boldsymbol{\nabla}(\mathbf{m}\cdot\mathbf{B}) - c^{-1}\mathbf{m}\times\partial_t\mathbf{E}$ so that

$$-2\int_0^t dt \int_a \mathbf{f}\cdot d\vec{\ell} = c^{-1}\oint_C \mathbf{m}\times\mathbf{E}\cdot d\vec{\ell} = \hbar\,\Delta\phi_{AC}$$

as expected.

We stress again the fact that, although for the AB effect we have established the above relationship between $\Delta\phi_{AB}$ and the integral expression of $\mathbf{f} = -(q/c)\partial_t\mathbf{A}$, this property does not imply that in the Aharonov-Bohm effect there is a force acting on the beam of charged particles [23]. In fact, $\mathbf{f}_{AB} = q\mathbf{E} = -(q/c)\partial_t\mathbf{A} = 0$ in the AB effect and in all the related tests or experiments performed. However, since $\Delta\phi_{AB}$ is linked to the variation $\mathbf{A} - \mathbf{A}^0$, with our semi-classical approach it is possible to express $\Delta\phi_{AB}$ also in terms of the time integral of $\mathbf{f}$, the force that would act on the beam of charged particles if the intensity of the current in the solenoid were made to vary during the time t from the value corresponding to $\mathbf{A}^0$ to the final value $\mathbf{A}$.

3 New field-free effects of the AB type

After having pointed out that the quantity measured in all the AB effects is not the phase shift $\delta\phi$ but the relative phase shift $\Delta\phi$ of equations (4, 5, 8, 9), on account of the above considerations on phase or gauge invariance, we can now foresee new field-free effects of the AB type that have not previously been considered in the literature.

3.1 Field-free effect for beams of charged particles with opposite signs

Since we are concerned mainly here with the theoretical aspects involved, let us apply the above considerations to the AB effect of Figure 2 for charges of opposite signs $\pm q$ in states $|a\rangle$ and $|b\rangle$ and traveling on one side of the solenoid on path a. Using expression (4), a phase or gauge transformation yields

$$\begin{aligned}\Delta\phi_{\pm} &= \frac{2q}{\hbar c}\left(\int_a (\mathbf{A}-\boldsymbol{\nabla}\chi)\cdot d\vec{\ell} - \int_a (\mathbf{A}^0-\boldsymbol{\nabla}\chi)\cdot d\vec{\ell}\right)\\ &= \frac{2q}{\hbar c}\int_a \left(\mathbf{A}-\mathbf{A}^0\right)\cdot d\vec{\ell} = \Delta\phi_{AB}\end{aligned}$$

so that $\Delta\phi_{\pm}$ cannot be gauged to zero. Thus, contrary to general belief, an AB effect for beams of charged particles with opposite signs is physically possible in principle, at least as far as gauge (or phase) invariance is concerned.

The description of the experimental conditions necessary for the test of an effect of this type goes beyond the scope of this paper. Additional investigations may be needed to develop realistic tests of this effect where beams of charges of opposite signs $\pm q$ traveling on the same path are involved.

Ideally, beams of charged particles with opposite signs may be realized with electrons and positrons, or with positively and negatively ionized atoms or molecules. Since two charged particles of opposite signs tend to attract (and, even worse, electrons and positrons tend to annihilate) these beams would be highly unstable, and there are a number of possible sources of disturbance, like fluctuations in local intensity, that produce instability in this kind of scenario. In order to achieve stability it is necessary to invoke external stabilizing fields. Consider for example a set of several long, parallel, linear charge densities of opposite sign lying on a plane. Each positively charged line lies in the middle of two negatively charged lines (and vice-versa), with all of them thus in electromechanical equilibrium. Two adjacent central lines of this set of parallel lines could mimic the two beams of charges of opposite signs for the test of this quantum effect and arrangements of this type may be used to minimize the instability of the beams, subject of course to the limitations imposed by Earnshaw's theorem [24].

Moreover, although electrons and positrons tend to annihilate, the probability of annihilation is not infinite. After annihilation of part of the beams, adjacent intense beams of electrons and positrons may result in beams of reduced intensity with a finite time of flight that might remain stable over short distances. If the time of flight is long enough, the distance may reach lengths useful for interferometric purposes. Furthermore, in the case of positively and negatively ionized atoms or molecules, the time of flight and corresponding length for stable interfering beams should be considerably greater. In fact, due to the greater inertia of massive atoms or molecules, these may travel for a longer time before the electrostatic attraction produces collision of opposite charges and the beam's stability is disrupted. Beams of heavy positive and negative ions and molecules are routinely used in cyclotron laboratories and beams of them can be accelerated at a wide range of energies.

For beams of particles [(+) and (−)] with charges of opposite signs $\pm q$, the following approach for testing this effect should be viable, at least conceptually. Interferometers for electrons or atoms exist already, and it should be possible to adapt them to positrons or ionized atoms and molecules. The two interfering beams (+) and (−), which ideally proceed from a common origin, could be prepared by a process that may employ the technique of beam-splitting. It is possible that they need not be superimposed and may travel slightly separated on parallel paths in the presence of the same external, field-free em interaction related to $\mathbf{Q}$ (or $\mathbf{A}$). Assuming that an interferometric pattern can be created, the quantity $\delta\phi_{\pm} = \phi_{+} - \phi_{-}$ represents the relative phase shift of the two [(+) and (−)] beams. Per se, this phase shift is not physically useful. However, if $\delta\phi_{\pm}$ can be measured for a value of the parameter $\mathbf{Q}$ (or $\mathbf{A}$) and, if the interferometric device is such that during the experiment $\delta\phi^0_{\pm}$ can be measured for a different value $\mathbf{Q}^0$ (or $\mathbf{A}^0$), the variation $\delta\phi_{\pm} - \delta\phi^0_{\pm}$ represents the sought-for phase shift variation $\Delta\phi_{\pm}$. Even a qualitative observation of $\Delta\phi_{\pm} = \delta\phi_{\pm} - \delta\phi^0_{\pm}$ would prove the observability of this quantum effect. In fact, if all the other parameters and physical conditions are the same during the experiment, the variation $\Delta\phi_{\pm}$ is due solely to the change $\mathbf{Q} - \mathbf{Q}^0$ of the relevant em interaction.

Considering the approaches used for detecting the phase variations of beams of magnetic dipole moments $\pm\mathbf{m}$ [15] and of electric dipoles $\pm\mathbf{d}$ [10], we can infer that if a system of beams of particles can be prepared in a superposition of states $|a\rangle$ and $|b\rangle$ of opposite em properties, the corresponding time-dependent wave function reads [10]

$$\Psi(t) = |a\rangle\cos(\phi_{em}t/t_0) + i\,|b\rangle\sin(\phi_{em}t/t_0). \qquad (10)$$

Here, the quantity ϕ_{em} represents the phase of the em interaction under consideration and the states $|a\rangle$ and $|b\rangle$ correspond to particles with opposite em properties, such as charges of opposite signs or particles that possess a magnetic or electric dipole moment with opposite orientation. For beams of particles with opposite magnetic dipole moments $\pm\mathbf{m}$ [15], the techniques used to prepare the states $|a\rangle$ and $|b\rangle$ are described more fully in reference [25]. In this case, the phase difference of the AC effect was measured using the Ramsey method of separated oscillatory fields [26]. The same method is being proposed [10] for detecting the phase of electric dipoles $\pm\mathbf{d}$. The details of the preparation of states $|a\rangle$ and $|b\rangle$ for electric dipoles is given in references [10,27]. The Ramsey loop acts as a beam splitter [15] providing the required superposition of $\pm\mathbf{m}$ or $\pm\mathbf{d}$, while the corresponding phase evolves between the two states.

We point out here the instance when the interferometric approaches [10,15] discussed above should be transposable to the case of a beam in states with charges of opposite signs $\pm q$, such as a beam of ionized atoms or molecules. The wave function (10) implies an evolution between states $|a\rangle$ and $|b\rangle$ while the em interaction is acting and measurement of the phase variation corresponding to the change $\mathbf{Q}-\mathbf{Q}^0$ is being made. Thus, the initial state may correspond to a beam of particles in the state of negatively ionized atoms or molecules. Due to a convenient interaction with an external agent, the beam may be electron-stripped in a way that, acting as beam splitter, transforms it roughly into the required superposition of $\pm$ ionized atoms or molecules. The particles travel in this state for suitable distance while the phase is being accumulated. In this case, techniques analogous to the ones described above and used for particles with opposite magnetic dipole moments $\pm\mathbf{m}$ [15] or for detecting the phase of electric dipoles $\pm\mathbf{d}$ [10], could be conveniently adapted to test the proposed AB type effect for beams of charges of opposite signs $\pm q$.

The alternatives explored above are preliminary tentative suggestions only, as several technical aspects and difficulties need to be considered and overcome before being able to obtain an appropriately coherent superposition of states involving beams of charged species of opposite signs $\pm q$. In closing, although the technology and interferometry for the test of this effect are not completely available at the moment, it is worth recalling that not long time ago the technology and interferometry for beams of particles with opposite magnetic dipole moments $\pm\mathbf{m}$ or electric dipoles $\pm\mathbf{d}$ was likewise unavailable, but which is today a reality [10,15]. Thus, although it is certainly a challenging experiment, discussions on this subject may act as a stimulating catalyst for further research and one cannot rule out the possibility that technological advances will make this quantum effect testable. We believe that a step in this direction has been made by showing above that, at least in principle and as far as gauge invariance requirements are concerned, this effect is physically feasible.

3.2 Field-free effect for beams of particles with opposite magnetic dipole moments $\pm\mathbf{m}$

Before concluding, we revise the AC effect and consider the feasibility of a field-free quantum effect for magnetic dipoles. Since, for the AC effect $\mathbf{Q}_{AC} = \mathbf{m}\times\mathbf{E}/c$, it would seem impossible to have a field-free ($\mathbf{E}=0$) quantum effect for particles possessing an intrinsic magnetic dipole $\mathbf{m}$. However, in reference [8] a generalized quantum phase for the magnetic dipole has been derived together with a field-free quantum effect for a beam of magnetic dipoles encircling a singularity. As shown below, the generalized Lagrangian for the magnetic dipole has been derived from that of an electric dipole by applying Maxwell's duality.

The application of $\mathfrak{L}_q$ to the two charges of the dipole $\mathbf{d}=q\mathbf{a}$ yields, in the dipole approximation with $\dot{\mathbf{d}}=0$, scalar potential $V=0$, and within other approximations described in reference [6],

$$\mathfrak{L}_d = K + \mathbf{v}\cdot\mathbf{Q} = K + \frac{1}{c}\mathbf{v}\cdot[(\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{A}]. \qquad (11)$$

The field-free effect for the electric dipole has been discussed in reference [7]. Other quantum effects for the electric dipole are described in references [4,5].

In the case of the magnetic-electric dipole interaction and in the absence of true free electric charges, the em momentum $\mathbf{Q}_{em}$ in equation (1) may be generalized [8] to $\mathbf{Q}_{em} = (1/4\pi c)\int(\mathbf{D}\times\mathbf{B})d^3x'$ where $\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$ and $\mathbf{P}$ is the polarization density. In order to derive a generalized Lagrangian for $\mathbf{m}$ that complies with equation (8), we use the em Maxwell duality described in reference [28] and considered also in reference [10]. Maxwell's duality transformations imply $\mathbf{d}\Rightarrow\mathbf{m}, \mathbf{E}\Rightarrow\mathbf{B}$, and $\mathbf{m}\Rightarrow-\mathbf{d}, \mathbf{B}\Rightarrow-\mathbf{E}$. In the dipole approximation, since $\mathbf{A}=\mathbf{A}_m=\mathbf{m}\times\mathbf{r}_m/r^3$ and $\mathbf{A}_d=\mathbf{d}\times\mathbf{r}_d/r^3$, the duality implies also $\mathbf{A}_d\Rightarrow\mathbf{A}$ and $\mathbf{A}\Rightarrow-\mathbf{A}_d$, and the Lagrangian of the magnetic dipole dual of (11) reads,

$$\mathfrak{L}_m = K - \frac{1}{c}\mathbf{v}\cdot[(\mathbf{m}\cdot\boldsymbol{\nabla})\mathbf{A}_d], \qquad (12)$$

where

$$\begin{aligned}-(\mathbf{m}\cdot\boldsymbol{\nabla})\mathbf{A}_d &= \mathbf{m}\times\boldsymbol{\nabla}\times\mathbf{A}_d - \boldsymbol{\nabla}(\mathbf{m}\cdot\mathbf{A}_d)\\ &= \mathbf{m}\times\mathbf{E} - \boldsymbol{\nabla}(\mathbf{m}\cdot\mathbf{A}_d),\end{aligned}$$

being $\mathbf{D}=\mathbf{E}=\boldsymbol{\nabla}\times\mathbf{A}_d$ and $\mathbf{H}=\mathbf{B}$ at the position of the magnetic dipole $\mathbf{m}$.

The Lagrangians $\mathfrak{L}_m$ discussed in the literature (Ref. [2,4,21]), contain only the term $\mathbf{Q}_m = c^{-1}\mathbf{m}\times\mathbf{E}$, where $\mathbf{E}$ is the electric field produced by true free electric charges. As pointed out in reference [8], in the special case of the magnetic-electric dipole interaction, there are two strong reasons to consider the $\mathfrak{L}_m$ of equation (12) as the correct one for $\mathbf{m}$: Maxwell's duality and the compliance with equation (8).

The field-free effect for magnetic dipoles, dual of the S effect [7] for electric dipoles, consists of beams of moving magnetic dipoles, possessing opposite magnetic dipole moments $\pm\mathbf{m}$, in a coherent superposition of states $|a\rangle$ and $|b\rangle$ that interact with the potential $\mathbf{A}_d$ of a distribution of electric dipoles with polarization density d and thickness τ, shown in Figure 3. The field $\mathbf{E}=\mathbf{D}$ due to the polarization $\mathbf{P}$ vanishes at the position of the magnetic dipole $\mathbf{m}$, i.e., $\mathbf{E}=\mathbf{D}=0$ on the path of the particles and we are dealing with a field-free quantum effect.

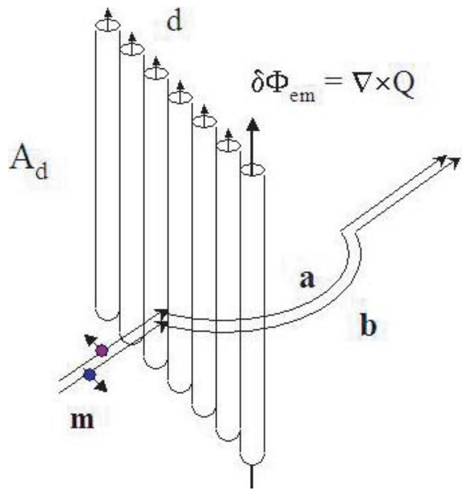

Fig. 3. Field-free effect for magnetic dipoles $\mathbf{m}$ in the presence of the vector potential $\mathbf{A}_d$ of a semi-plane of electric dipoles $\mathbf{d}$. The particles of the two beams in a coherent superposition of states $|a\rangle$ and $|b\rangle$ have opposite magnetic dipole moments $\pm\mathbf{m}$, parallel to $\mathbf{v}\times\mathbf{d}$, and the classical paths of the particles lie entirely at one side of the singularity, the em flux tube $\delta\Phi_{em}$. This effect is the Maxwell dual $\mathbf{d}\Rightarrow\mathbf{m}$ of the effect for electric dipoles (Ref. [7]) interacting with the vector potential $\mathbf{A}$ of a semi-plane of magnetic dipoles.

Taking the moment $\pm\mathbf{m}$ of the particle parallel to the $\mathbf{v}\times\mathbf{d}$ direction, calculations dual to those of reference [7] yield

$$\mathbf{Q}_m = -c^{-1}(\mathbf{m}\cdot\boldsymbol{\nabla})\mathbf{A}_d = c^{-1}2md\tau\boldsymbol{\nabla}\theta = L\boldsymbol{\nabla}\theta,$$

and the relative phase shift reads

$$\Delta\phi = -\frac{2}{\hbar c}\oint_a(\mathbf{m}\cdot\boldsymbol{\nabla})\mathbf{A}_d\cdot d\mathbf{x} = \frac{4\pi md\tau}{c\hbar}. \qquad (13)$$

This effect may be observed in magnetic moment interferometry and its verification is within reach of present experimental technique (Ref. [8]).

4 Conclusions

In conclusion, the criteria (1) and (2) of Section 2 indicate that $\Delta\phi$ cannot be removed by a phase or gauge transformation even in the case of particles that travel on one side of the singularity in a coherent superposition of states $|a\rangle$ and $|b\rangle$. The test performed for the AC effect [15] is an experimental confirmation of our theoretical analysis. The new field-free quantum effects presented in Section 3, for charged particles $\pm q$ or magnetic dipoles $\pm\mathbf{m}$ in a coherent superposition of states, are a consequence of these criteria and of the existence of the dual momentum $\mathbf{Q}_m = -c^{-1}(\mathbf{m}\cdot\boldsymbol{\nabla})\mathbf{A}_d$ of equation (12) for the electric-magnetic dipole interaction.

In the standard interpretation of the AB effect, the observable relative phase shift $\Delta\phi_{AB}$ is linked to the magnetic flux Φ_m of the solenoid, $\Delta\phi_{AB} = (q/c\hbar)\Phi_m$, encircled by the interfering beams of electrons. However, our results indicate that the interfering beams of particles need not encircle the singular em flux, so that some of the quantum effects of the AB type are not necessarily directly related to such em flux.

Instead, the underlying fundamental physical quantity that seems to be relevant and physically meaningful in all the effects of the AB type is the variation $\Delta\mathbf{Q}$ of the interaction momentum $\mathbf{Q}$. Although $\mathbf{Q}$ may still be linked to the em flux Φ_{em} defined in equation (2), its variation $\Delta\mathbf{Q}$ that appears in equations (4) and (5) is directly related to the variation $\Delta\mathbf{Q}_{em} = \mathbf{Q}_{em} - \mathbf{Q}^0_{em}$ of the linear momentum of the em fields. The momentum $\mathbf{Q}_{em}$ is an integral expression and can be related to a nonlocal quantity (such as the vector potential $\mathbf{A}$) but its phase and gauge invariant variation $\Delta\mathbf{Q}_{em}$ leads to the observable phase shift variation $\Delta\phi$ of equations (4–9).

The proposed field-free quantum effects of the AB type for interfering beams of particles with opposite charges or magnetic dipole moments, are in principle viable physical effects. Although some of the available interferometry should be transposable to the case of a beam (or beams) of ionized atoms or molecules, further technological advances are needed to make this quantum effect testable. Instead, the test of the effect for interfering beams of particles with opposite magnetic dipole moments that may be observed in magnetic moment interferometry, should be within reach of present experimental techniques.

I am indebted to George T. Gillies for helpful comments and revision of manuscript. This research was supported by a grant from the CDCHT (ULA, Mérida, Venezuela).

References

1. Y. Aharonov, D. Bohm, Phys. Rev. **115**, 485 (1959)
2. Y. Aharonov, A. Casher, Phys. Rev. Lett. **53**, 319 (1984)
3. V.M. Tkachuk, Phys. Rev. A **62**, 052112-1 (2000)
4. J. Anandan, Phys. Rev. Lett., **85**, 1354 (2000)
5. X.G. He, B.H.J. McKellar, Phys. Rev. A **47**, 3424 (1993); M. Wilkens, Phys. Rev. A **49**, 570 (1994); M. Wilkens, Phys. Rev. Lett. **72**, 5 (1994); J. Yi, G.S. Jeon, M.Y. Choi, Phys. Rev. B **52**, 7838 (1995); C.R. Hagen, Phys. Rev. Lett. **77**, 1656 (1996); J. Anandan, Phys. Rev. Lett. **48**, 1660 (1982); J. Anandan, Phys. Lett. A **138**, 347 (1989); H. Wei, R. Han, X. Wei, Phys. Rev. Lett. **75**, 2071 (1995)
6. G. Spavieri, Phys. Rev. Lett. **81**, 1533 (1998); G. Spavieri, Phys. Rev. A **59**, 3194 (1999)
7. G. Spavieri, Phys. Rev. Lett. **82**, 3932 (1999)
8. G. Spavieri, Phys. Lett. A **310**, 13 (2003)
9. M. Peshkin, H.J. Lipkin, Phys. Rev. Lett. **74**, 2847 (1995)
10. J.P. Dowling, C.P. Williams, J.D. Franson, Phys. Rev. Lett. **83**, 2486 (1999)
11. D.G. Boulware, S. Deser, Phys. Rev. Lett. **63**, 2319 (1989); C. Fuchs, Phys. Rev. Lett. **42**, 2940 (1990); G. Spavieri, G.T. Gillies, M. Rodriguez, Metrologia **41**, S159 (2004)
12. R.G. Chambers, Phys. Rev. Lett. **5**, 3 (1960)
13. A. Tomonura et al., Phys. Rev. Lett. **56**, 792 (1986), and therein cited references
14. A. Cimmino et al., Phys. Rev. Lett. **63**, 380 (1989)

15. K. Sangster, E.A. Hinds, S.M. Barnett, E. Riis, Phys. Rev. Lett. **71**, 3641 (1993); K. Sangster, E.A. Hinds, S.M. Barnett, E. Riis, A.G. Sinclair, Phys. Rev. A **51**, 1776 (1995); see also R.C. Casella, Phys. Rev. Lett. **65**, 2217 (1990)
16. T.H. Boyer, Phys. Rev. D **8**, 1667 (1973)
17. X. Zhu, W.C. Henneberger, J. Phys. A **23**, 3983 (1990); these authors did not know the previous proof by Boyer
18. G. Spavieri, submitted, this author did not know the previous works by Boyer and by Zhu and Henneberger
19. M.V. Berry, Proc. R. Soc. Lond. A **392**, 45 (1984); see also B. Simon, Phys. Rev. Lett. **51**, 2167 (1983)
20. W. Dittrich, M. Reuter, *Classical and Quantum Dynamics* (Springer-Verlag, New York, 1994), 2nd edn., Chap. 28
21. Y. Aharonov, P. Pearle, L. Vaidman, Phys. Rev. A **37**, 4052 (1988)
22. G. Spavieri, Nuovo Cim. B **109**, 45 (1994); G. Spavieri, G.T. Gillies, Nuovo Cim. B **118**, 205 (2003)
23. For a semi-classical interpretation of the AB effect, see: T.H. Boyer, Phys. Rev. A **36**, 5083 (1987); G. Spavieri, G. Cavalleri, Europhys. Lett. **18**, 301 (1992)
24. S. Earnshaw, Trans. Camb. Phils. Soc. **7**, 97 (1842)
25. D. Cho, K. Sangster, E.A. Hinds, Phys. Rev. A **44**, 2783 (1991)
26. N.F. Ramsey, *Molecular Beams* (Oxford University Press, Oxford, 1956)
27. F. Biraben et al., Rev. Sci. Instrum. **61**, 1468 (1990); G.W.F. Drake et al., Phys. Rev. A **46**, 113 (1992)
28. J.D. Jackson, *Classical Electrodynamics*, 2nd edn. (Wiley, New York, 1975), Sect. 11